Use of an Inference technique for Sensitivity Analysis of RL parameters of Wound Inductors Extracted from the Finite Element Method

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***Abstract*—In this work, we compute the sensitivity of wound inductors RL parameters extracted using the Finite Element (FE) method with respect to geometric and material uncertainties, along a wide range of frequencies (from DC to 1 MHz). To that end, we compute the *Sobol* indices associated to a Polynomial Chaos Expansion based surrogate model of the uncertain FE model thanks to Bayesian inference technique.**

***Index Terms*—Finite element method, random input, surrogate model, Bayesian inference, wound inductors**

I. Introduction

Numerical models of a physical system almost always depend on uncertain parameters. For instance, wound inductors are characterized by material (i.e. material properties which are not known precisely) and geometric (i.e. conductors positions in the winding window) uncertainties. A correct modelling of such uncertainties, as well as a good understanding of the sensitivity of physical quantities w.r.t uncertainties, is fundamental for ensuring an optimal design of the component. The Finite Element (FE) method can be employed to capture these uncertainties in RLC models using a Monte Carlo simulation approach [1]. However, performing a sensitivity analysis with such a model is very CPU intensive. In this work, we develop a stochastic surrogate model of wound inductors based on chaos polynomial expansion (PCE), in order to mitigate this problem. We then compute the *Sobol* indices [2], [3] over a wide range of frequencies (from DC to 1 MHz) in order to quantify the sensitivity of the PCE surrogate.

II. Methodology

The overall methodology is built upon a 2D magnetodynamic FE model [4] for the extraction of RL parameters, in combination with an original algorithm mimicking the manual wounding operation [5] in order to model the uncertain positions of conductors in the winding window. It is depicted in Fig. 1. We compute a PCE surrogate in order to reduce the number of numerical model calls needed to perform a Monte Carlo simulation (with a brute FE model). In addition to this, we compute the *Sobol* coefficients from the PCE surrogate which does not require any additional evaluation of the deterministic model (which is expensive in terms of resources and computation time) once it has been built.

According to this chart, N_{iter} is the number of iterations needed to build the PCE surrogate from distributions of the reduced random input vector. N_{iter} is limited by N_{max} which is linked to the dimension of random input vector [5]. In our case, without the homogenization process which allows us to transform geometrical uncertainties into material ones, this dimension can easily reach an order of magnitude close to 100.

The uncertainties taken into account in this study arise from random positions of conductors in the winding window and from material properties of ferrite core (magnetic permeability $\underline{\mu}_{core}$ and electrical conductivity σ_{core}). The complex form of material properties allows to take into account in a natural way the electrical and magnetic losses in finite element formulations.

Due to the high number of Monte Carlo iterations with an embedded time-consuming FE based model, especially with regard to the high number of geometric uncertainties, the reduction in the dimension of the problem is achieved through the transformation of geometrical uncertainties (positions of conductors) into material ones (equivalent magnetic reluctivity $\underline{\nu}_{prox}$ related to the proximity effect and equivalent impedance $\underline{Z}_{skin}$ related to the skin effect) via homogenization technique [6].

A 2D magneto harmonic formulation with massive conductors and appropriate limit conditions on an optimal RVE (Representative Volume Element) [7] in the winding, allow to extract the desired equivalent properties (electric conductivity and magnetic reluctivity for instance). Finally, the dimension of the random input is reduced to 7 components, among which we find:

- $X_1 = \mathfrak{Re}\left(\underline{Z}_{skin}\right)$, image of losses due to skin effect phenomena in the winding;
- $X_2 = \mathfrak{Im}\left(\underline{Z}_{skin}\right)$, image of the reactive power transmitted by the winding;
- $X_3 = \mathfrak{Re}\left(\underline{\nu}_{prox}\right)$, image of the energy associated with the proximity effect in the winding;

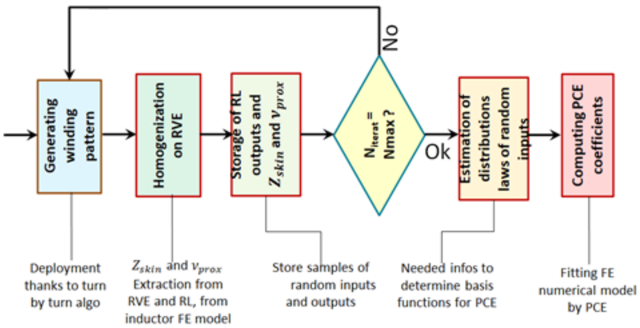

Fig. 1. Process of PCE surrogate construction.

- $X_4 = \mathfrak{Im}\left(\underline{\nu}_{prox}\right)$, image of losses linked to the proximity effect between conductors;
- $X_5 = \mathfrak{Re}\left(\underline{\nu}_{core}\right)$, image of the magnetic energy in the ferrite core;
- $X_6 = \mathfrak{Im}\left(\underline{\nu}_{core}\right)$, image of the ferrite core losses;
- $X_7 = \sigma_{core}$, image of losses due to the eddy current in the ferrite core. It is a scalar depending on the imaginary part of the electric permittivity $\underline{\epsilon}_{core}$.

This dimension reduction of the random vector input is benefit for sensitivity analysis of the model i.e. when studying the behaviour of a numerical model due to the random variability of input parameters. This kind of analysis may be computationally intensive when using Monte Carlo simulation to compute *Sobol* coefficients [3] which allow to determine the influence of random inputs on outputs of the model. To deal with this difficulty, we use a stochastic surrogate model based on PCE. This is due to one of the important properties of PCE, that of containing information on their ANOVA (Analysis Of Variance) decomposition.

For the computation of the PCE surrogate model, we need to know distribution law of the obtained reduced random input vector. This problem is associated to the inverse problem where unknown parameters (related to distributions) are estimated based on experimental data which are indirectly associated with these parameters through a computational model [8]. This problem can be solved using Bayesian methods and particularly the Bayesian inference technique. This technique consists of backward propagation of information about observations in order to extract the distribution of the model inputs. For instance, in statistical inference, one considers that the data are made of independent realizations of an underlying random vector and an assumption on the shape of the probability density function PDF (Weibull, Gaussian, log-normal, etc.). The used tools here are based on UQLab [2] which is a framework that can be plugged into Matlab to perform uncertainty quantification.

III. Case Study

In this work, we analyse a typical commercial inductor (MCSCH895-680KU, as illustrated in Fig. 2). It is made up of a NiZn ferrite core type, with a 44-turns winding distributed over 4 layers. The first three layers include 12 turns

Fig. 2. Illustration of the analysed component.

per layer. The conductor diameter is 0.37 mm. According to the manufacturer, this inductor is characterized by a nominal inductance value of 680 μH $\pm$ 10%. The material properties of the ferrite core are modelled using a *Debye* relaxation model along the frequency range of study [5]. In this model, the random aspect is carried out by static quantities for which the manufacturer often gives the relative error (20% for magnetic permeability and 10% for electrical conductivity). The dimension of the random input vector (set of geometrical and material uncertainties) of the numerical model of the studied inductor has been reduced to 7, among which, real and imaginary parts of $\underline{\nu}_{prox}$, $\underline{Z}_{skin}$ and $\underline{\mu}_{core}$ and the ferrite core conductivity σ_{core}. Results of sensitivity analysis will be presented in the full paper.

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